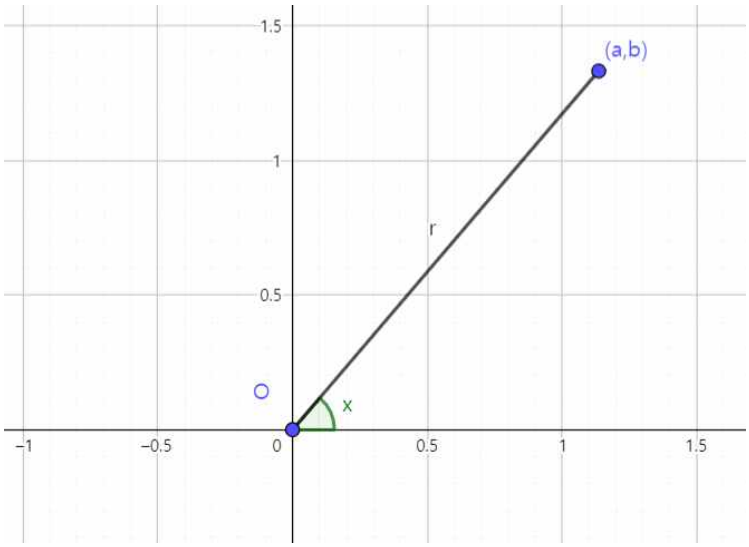
Trigonometric Functions

삼각함수 by Scubed

1. 삼각함수의 정의



$$\sin x = \frac{a}{r}, \cos x = \frac{b}{r}, \tan x = \frac{a}{b}, \ \tan x = \frac{\sin x}{\cos x}$$

$$\csc x = \frac{1}{\sin x}, \sec x = \frac{1}{\cos x}, \cot x = \frac{1}{\tan x}$$

2. 삼각함수의 제곱 공식

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

3. 삼각함수의 대칭성

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

4. 삼각함수의 덧셈정리

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

5. 삼각함수의 반각공식

$$\sin^2\frac{x}{2}=\frac{1-\cos x}{2}$$

$$\cos^2\frac{x}{2}=\frac{1+\cos x}{2}$$

$$\tan^2\frac{x}{2}=\frac{1-\cos x}{1+\cos x}$$

6. 삼각함수의 n배각공식

$$\sin nx=\sum_{k=0}^{n}{}_nC_k\cos^{n-k}x\sin^k x\sin\frac{k\pi}{2}$$

$$\cos nx=\sum_{k=0}^{n}{}_nC_k\cos^{n-k}x\sin^k x\cos\frac{k\pi}{2}$$

7. 삼각함수의 합-곱 변환 공식

$$2\sin x\cos y=\sin(x+y)+\sin(x-y)$$

$$2\cos x\sin y=\sin(x+y)-\sin(x-y)$$

$$2\cos x\cos y=\cos(x+y)+\cos(x-y)$$

$$2\sin x\sin y=\cos(x+y)-\cos(x-y)$$

$$\sin x+\sin y=2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x-\sin y=2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\cos x+\cos y=2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x-\cos y=-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

8. 삼각함수의 합성

$$a\sin x+b\cos x=\sqrt{a^2+b^2}\sin(x+\alpha),\ \tan\alpha=\frac{b}{a}$$

$$a\sin x+b\cos x=\sqrt{a^2+b^2}\cos(x-\beta),\ \tan\beta=\frac{a}{b}$$

9. 삼각함수의 곱과 2배각

$$\sin(x+y)\sin(x-y)=\frac{\cos2y-\cos2x}{2}$$

$$\csc(x+y)\csc(x-y)=\frac{2}{\cos2y-\cos2x}$$

$$\cos(x+y)\cos(x-y)=\frac{\cos2y+\cos2x}{2}$$

$$\sec(x+y)\sec(x-y)=\frac{2}{\cos2x+\cos2y}$$

$$\tan(x+y)\tan(x-y)=\frac{\cos2y-\cos2x}{\cos2y+\cos x}$$

$$\cot(x+y)\cot(x-y)=\frac{\cos2x+\cos2y}{\cos2y-\cos2x}$$

10. 삼각함수의 제곱의 합과 2배각*

$$\sin^2(x+y)+\sin^2(x-y)=1-\cos2x\cos2y$$

$$\cos^2(x+y)+\cos^2(x-y)=1+\cos2x\cos2y$$

$$\tan^2(x+y)+\tan^2(x-y)=\frac{2(\sin^2 2x+\sin^2 2y)}{(\cos2x+\cos2y)^2}$$

$$\csc^2(x+y)+\csc^2(x-y)=\frac{4(1-\cos2x\cos2y)}{(\cos2x-\cos2y)^2}$$

$$\sec^2(x+y)+\sec^2(x-y)=\frac{4(1+\cos2x\cos2y)}{(\cos2x+\cos2y)^2}$$

$$\cot^2(x+y)+\cot^2(x-y)=\frac{2(\sin^2 2x+\sin^2 2y)}{(\cos2x-\cos2y)^2}$$

11. 삼각함수의 특수한 항등식*

$$\sin^2x+\sin^2y+\cos^2(x+y)+2\sin x\sin y\cos(x+y)=1$$

$$\cos^2x+\cos^2y-\cos^2(x+y)-2\sin x\sin y\cos(x+y)=1$$

$$\sin^2x+\sin^2y-\cos^2(x+y)+2\cos x\cos y\cos(x+y)=1$$

$$\cos^2x+\cos^2y+\cos^2(x+y)-2\cos x\cos y\cos(x+y)=1$$

$$\sin^2x+\cos^2y+\sin^2(x+y)-2\sin x\cos y\sin(x+y)=1$$

$$\cos^2x+\sin^2y-\sin^2(x+y)+2\sin x\cos y\sin(x+y)=1$$

12. 삼각함수의 극한

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{\tan x}{x}=1$$

13. 삼각함수의 미분

$$(\sin x)'=\cos x$$
$$(\cos x)'=-\sin x$$
$$(\tan x)'=\sec^2 x$$
$$(\sec x)'=\tan x\sec x$$
$$(\csc x)'=-\cot x\csc x$$
$$(\cot x)'=-\csc^2 x$$

14. 삼각함수의 적분

$$\int \sin x dx=-\cos x+C$$

$$\int \cos x dx=\sin x+C$$

$$\int \tan x dx=-\ln(\cos x)+C$$

$$\int \sec x dx=\ln(\tan x+\sec x)+C$$

$$\int \csc x dx=-\ln(\cot x+\csc x)+C$$

$$\int \cot x dx=\ln(\sin x)+C$$

15. 삼각함수의 테일러 전개

$$\sin x=\frac{x}{1!}-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots=\sum_{k=1}^{\infty}(-1)^{k-1}\frac{x^{2k-1}}{(2k-1)!}$$

$$\cos x=\frac{1}{0!}-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots=\sum_{k=1}^{\infty}(-1)^{k-1}\frac{x^{2k}}{(2k)!}$$